THERMODYNAMICS OF RIVERS

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Contents

Summary

This chapter deals with thermal regime of rivers and therefore the basic thermodynamic relations for river waters are described. At first a detailed description of the fundamental laws of thermodynamics applied to rivers is given and then some examples of how to use these laws to solve thermal problems are presented. Description of the first law of thermodynamics requires introducing some fundamental variables like the internal energy, conduction, convection, enthalpy, and so on. All these thermodynamic variables are described in detail. One example is related to the thermal problem of mixing two rivers of different temperatures and others are related to a case of free or natural convection. These examples are often met in environmental engineering practice.

The most important issue in river thermodynamics is the heat exchange through the water-air interface. The physical processes involved in the heat transfer through a water surface are extremely complex and dependent upon a variety of hydrodynamical and

meteorological factors. The following processes are described: the net solar radiation flux, the net atmospheric radiation flux, the water surface radiation flux, the evaporation heat flux and the sensible heat flux. These heat fluxes are defined and some approximation formulae to calculate them are given. Also the problem of the heat exchange between water and river bed is briefly discussed.

A case study is presented. It comprises two engineering problems related to calculation of depth-averaged temperature in rivers. The first one is related to the calculation of the temperature field in a given river reach downstream of an outlet of a thermal discharge. The river reach is wide and shallow with bends of moderate curvature, where centrifugal forces are negligibly small. Flow is steady and both the river depth and the depth-averaged velocity can be varied in both horizontal directions. The main mechanism that determines the river temperature is the mixing of discharge warm waters into a river. The second problem is related to calculation of the water temperature field of a given river reach under extreme meteorological conditions. These conditions (three consecutive hottest days) are chosen based on historical data. The main mechanism that determines the river temperature, in this case, is surface heat exchange through the water-air interface. This mechanism is discussed and the problem of calculation of river temperature distribution in these extreme conditions is solved and the results are discussed in detail.

1. Introduction

The aim of this chapter is to present the basic thermodynamical properties of river waters. These properties are related to the river temperature, which changes with time and space. Knowledge of the river temperature and its changes is very important for some environmental problems as well as for technical ones, e.g. related to constructing or rebuilding of thermal power plants. From the environmental point of view the river temperature has great influence on water quality and on the whole aquatic life. It varies according to a seasonal rhythm upon which the daily changes are superimposed. Also, the water temperature is closely linked to local climate conditions. The temperature of a river flowing slowly through a plain depends on local meteorological conditions. The surface exchange of heat between water and atmosphere is a main factor of formation of thermal regime in these rivers.

The temperature of some rivers is disturbed by a thermal power plant located near a river. This situation is met very often today. Our rivers have been used as coolers for heated water from steam-electric power plants. Modern steam-electric power plants discharge approximately 1.5 – 2 kWh of waste heat for every kWh of electrical energy produced. Traditionally, this heat has been discharged into a water body adjacent to the power plant using ‘once-through” cooling. With this method water is pumped through the condensers, where its temperature is increased and then discharged back into the body of receiving water. Discharge of hot water into rivers involves two different approaches to study the thermal regime of a river. The “near field” problem is concerned with a very limited zone of river near the point of heat discharge. The temperature distribution in the near field (not very far from the discharge) depends mainly on the initial distributions of velocity and temperature or more precisely on differences in velocities and temperatures between the discharged and river waters.

These differences create two mechanisms of mixing: entrainment coming from different velocities and the buoyancy coming from different temperatures. Besides there are occurring natural mechanisms of mixing: advection and diffusion. These mechanisms are described in *Constituent Transport*. The roles of particular mechanisms depend on specific situation. In "far field" problems, the main emphasis is placed on large-scale effects. The effect of an initial perturbation on the thermal regime of a river, in a considerable distance downstream of the disturbance, is rather limited. In far field the two natural mechanisms and the heat exchange through the air-water interface are the main factors, which create the thermal regime of rivers. Of course, the location of the boundary between these two fields creates a further problem.

The first and second laws of thermodynamics are fundamental principles of classical physics that determine all thermal processes of our environment. The second law says that heat flows from high to low temperature, i.e., in the direction of decreasing temperature. In principle, this law is not used in analysis of heat transfer problems. In contrast, the first law of thermodynamics, also known as the law of conservation of energy, is the fundamental relation in solving any problem of heat transfer. Almost all thermal river problems are concerned with the determination of the one-dimensional, two-dimensional or three-dimensional temperature distributions over a river reach. To determine these distributions the conservation energy law is used together with the mass and momentum conservation laws. All these conservation principles are described in detail in this chapter. The application of these principles in a general situation is very difficult. This difficulty arises primarily from mathematics and physics. There are no suitable numerical algorithms solving the general equations of hydro- and thermo-dynamics. Also, there are no suitable formulae for description of many physical processes, e.g. the heat losses due to the process of evaporation. These shortages force us to make some simplifications related to our physical approach as well as the mathematical approach. These simplifications allow us to define the mathematical model as a set of equations (usually differential equations), describing the temperature field in a given river reach, which can be solved.

To present the analysis of the thermal regime problem as a well posed problem, it is necessary to supplement the basic equations by appropriate initial and boundary conditions. These equations describe the physical, chemical and biological processes within the river body, while the boundary conditions reflect the constraints of the surrounding world acting upon the fluid motions. The number and type of initial and boundary conditions depend on the specific problem of interest and on the type of equations that are used. Therefore, the initial and boundary conditions in river thermal problem are of considerable variety. Usually in river problems the exchange energy and mass with the external world takes place only at the water surface. In this case, the boundary conditions should be prescribed at the water-air interface and the water-soil interface. Especially difficult are thermal boundary conditions at the water-air interface where the energy fluxes across the water surface should be described. There are many energy fluxes, which are traditionally used to describe the total heat exchange at water surface, namely the solar and atmospheric radiation fluxes, the water surface radiation flux, the evaporation heat flux and the sensible heat flux. Each of these fluxes is described by more or less empirical formulae with many empirical coefficients. Therefore, the accuracy of evaluation of heat exchange at the water-air interface is not very high.

Also the source/sink term in the energy balance equation is very difficult to evaluate; especially the term describing the chemical and biological processes. It is known from hydrobiological and limnological studies that algae and other "autotrophs" (macrophytes, photosynthetic bacteria) use solar radiation as their only energy sources, produce biomass from inorganic nutrients (nitrate, phosphate, silicate) and water. Thus they become primary producers of new organic matter on which the following levels of food web depend: herbivorous zooplankton and zoobenthos, carnivorous zooplankton and zoobenthos, small fishes, predators. The passage of energy through the food web causes that every member of the food web receives energy from the preceding "level", partly using for vital functions and partly supplying energy to the next "level". The amount of energy accumulated at each level is small compared with the flow of energy through the air-water interface. This energy is not taken into account in the discussion of the energy balance equation.

2. The Basic Laws and Equations of Thermodynamics

There are three laws of thermodynamics. The third law is related to the behavior of the entropy difference at the absolute zero temperature and it is not applied to river problems. To solve any thermal problem related either to natural rivers (without artificial sources of heat) or heat loaded rivers the first and second laws of thermodynamics are to be considered. Both laws are of significance in thermodynamics but only the first thermodynamics law is widely used to solve many engineering problems. The second law can be called the principle of the increase of entropy and it can be formulated in the form of Clausius statement: Heat can never pass from a colder to a warmer body without some other change occurring at the same time. It is necessary that an approach, which is used to solve any thermal problem, must obey the second law of thermodynamics (entropy principle). Furthermore, the entropy and the entropy production depend on practically all hydro-physical, hydro-chemical, and hydro-biological variables and processes of the river system. Therefore, by the entropy principle the many phenomenological relationships are reduced to a few important equations and basic principles. The first law can be written in the form of an energy balance expressing the fact that the sum of the energy fluxes entering the control volume through the boundaries must be equal to the increase of energy contained in the fluid filling the control volume.

$$\frac{dE}{dt} = \frac{dQ}{dt} + \frac{dW}{dt} \qquad (1)$$

Where

E is the total energy in J,
Q is the all heat transfer across the boundary of the control volume and sources of heat generating within the control volume in J,
W is the total work done by all forces exerting on the control volume in J,
t is time.

The left side of the above equation denotes the rate of energy stored in the fluid within the control volume. The term dQ/dt summarizes all heat transfer rate across the boundary and sources generating heat within the control volume in unit of time. The last term indicates the energy transfer to the fluid in the control volume as work done by all forces acting on the fluid. Equation (1) expresses the first law of thermodynamics and states that the total energy increase of the control volume equals to the heat received by the control volume plus the work done on the volume.

The total energy stored within the control volume is the sum of internal energy and kinetic energy, i.e.

$$E = \left(e_{\mathrm{w}} + \frac{U^2}{2} \right) \rho \; dV \equiv e \rho \, dV \tag{2}$$

where e_{w} is the internal energy per unit mass [L^2 T^{-2}], U is the average control volume velocity, ρ is density of water [M L^{-3}], dV is a volume of the control element [L^3] and e is a total energy of the unit of mass in J kg^{-1}.

The rate of change of the total energy following the fluid motion is

$$\frac{dE}{dt} = \frac{\partial}{\partial t} \left(\rho \, dV \, e \right) + \frac{\partial}{\partial x_i} \left(\rho \, dV \, e u_i \right) \tag{3}$$

The total heat added to the control volume fluid is due to the intensity of generation or dissipation of heat q^* per unit volume plus the net heat conduction q^{c}, i.e.

$$\frac{dQ}{dt} = q^* \, dV - \frac{\partial}{\partial x_i} q_i^{\mathrm{c}} \; dV \tag{4}$$

where q_i^{c} is the i-component of the heat conduction flux per unit area and time in J m^{-2} s^{-1}.

Beginning with Eq. (4) the so-called summation convention has been adopted. This convention can be formulated as follows: if in an expression a lower-case subscript appears twice, it is understood that in this expression summation with respect to this subscript occurs, and the latter assumes the values 1, 2, 3.

Finally, the net work done on the control element is due to pressure, shear and gravity forces, i.e.

$$\frac{dW}{dt} = -\frac{\partial}{\partial x_i} \left(u_i p \right) \; dV + \frac{\partial}{\partial x_j} \left(u_i \tau_{ij} \right) \; dV + \rho \, u_i \; g_i dV \tag{5}$$

Where shear stress tensor is defined as

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad (6)$$

The positive signs of forces (stresses and body force) in Eq.(5) are chosen according to the rules defined in textbooks of classical mechanics as well as in books on mass heat transfer. It was assumed in Eq. (5), that the work is positive when the forces act in the flow direction and negative when they act opposite to the flow direction.

The first law of thermodynamics can be written in more explicit form by inserting the above developed quantities (Eqs. (3), (4), and (5) into Eq. (1).

$$\frac{\partial}{\partial t}(\rho\, e) + \frac{\partial}{\partial x_i}(\rho u_i\, e) = q^* - \frac{\partial}{\partial x_i} q_i^{\mathrm{c}} - \frac{\partial}{\partial x_i}(u_i\, p) + \frac{\partial}{\partial x_j}(u_i \tau_{ij}) + \rho u_i g_i \qquad (7)$$

Eq.(7) is a very general form of the first law of thermodynamics and it has been used to solve the different problems of heat convection. One must keep in mind that any term in which the same index appears twice stands for a summation.

For river problems equation (7) can be written in a simpler form. The river flow velocities are rather small so they do not change the pressure. The total internal energy is replaced by the other thermodynamics function, the enthalpy that is defined for an irreversible process by formula:

$$h = e + \frac{p}{\rho} \qquad (8)$$

and for processes taking place at constant pressure $dh = c_{\mathrm{p}}\, dT$, c_{p} is the specific heat at constant pressure in [J kg^{-1} $^{\mathrm{o}}$K^{-1}].

The flux of heat transfer by conduction is described by Fourier’s law, i.e.

$$q_i^{\mathrm{c}} = -k\frac{\partial T}{\partial x_i} \qquad (9)$$

where k is the thermal conductivity [J m^{-1} s^{-1} K^{-1}]. Term $(u_i \tau_{ij})$ in Eq.(7) expresses the j-component of density of the energy flux related to friction processes and term $(\partial(u_i \tau_{ij})/\partial x_j)$ represents the dissipation rate of energy caused by shear stresses. For a Newtonian, incompressible fluid the rate at which the energy of unit volume is dissipated into heat can be written in the form:

$$\frac{\partial}{\partial x_j}(u_i \tau_{ij}) = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 \qquad (10)$$

Where
μ is the dynamic viscosity [N s / L^2] ≡ [Pa s]

Substituting (8), (9), and (10) into (7) and assuming that there is no energy sources or sinks (generation or dissipation) in the considered reach of river, gives:

$$\rho c_p \left(\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} \right) = \frac{\partial}{\partial x_i} \left(k \frac{\partial T}{\partial x_i} \right) + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 + \rho u_i g_i \tag{11}$$

The last two terms in Eq. (11) express the dissipation of energy due to shear stresses and gravitational force respectively. Usually they are negligibly small compare to other terms and then Eq. (11) simplified to the form:

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i} D_{\mathrm{T}} \frac{\partial T}{\partial x_i} \tag{12}$$

where $D_{\mathrm{T}} = k/(\rho c_{\mathrm{p}})$ is thermal diffusivity in m^2 s^{-1}.

Eq.(12) can be deduced more easily from the continuity equation for heat, which comes from the more general law of conservation of heat. If it is assumed that the heat ($\rho\, c_{\mathrm{p}}\, T$) is transported only by convection ($\rho\, c_{\mathrm{p}}\, T\, u$) and thermal diffusion, then the continuity equation for heat gives Eq.(12). This equation can be used to calculate the temperature distribution in rivers or lakes when the velocity field is known.

If not, the water velocity can be obtained from the basic hydrodynamics equations, i.e., the conservation equation of mass and momentum. These equations, for constant density, are written below:

The conservation of mass equation is

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{13}$$

The conservation of momentum is

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\mu}{\rho} \frac{\partial^2 u_i}{\partial x_i \partial x_i} - g_i \tag{14}$$

Equations (14) are called the Navier-Stokes equations and together with continuity equation (13), and initial and boundary conditions they describe the pressure and velocity fields of rivers.

In the case when the temperature distribution in not homogenous, Equation (14) should be modified. This modification is shown in Section 4, on free or natural convection.

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Bibliography

Brutsaert, W. (1988). *Evaporation into the Atmosphere*. D. Reidel Publishing Company, Dordrecht. [This book provides a comprehensive overview of the all processes of evaporation]

Coulson, J. M. and Richardson, J. F. (1978). *Chemical Engineering*. Pergamon Press, Oxford. [It covers the properties of the particulate chemical engineering systems]

Eckert, E. R. and Drake, R. M. (1987). *Analysis of Heat and Mass Transfer*. Hemisphere Publishing Corporation, Berlin. [The core of the book is devoted to the boundary layer theory with special emphasis on the laminar and turbulent thermal boundary layer]

Encyclopedic Dictionary of Physics. Ed. Thewlis J. Pergamon Press 1962, Oxford, London, New York. [It gives the basic information on fundamental laws, hypotheses and ideas of physics]

Jacquet J. (1982). Simulation of the Thermal Regime of Rivers, in *Mathematical modeling of Water Quality: Streams, Lakes, and Reservoirs,* edited by Orlob G.T. pp.150 – 175. John Wiley & Sons, Chichester. [It presents the mathematical model of temperature distribution in rivers in details]

Jobson H.E. (1977). Bed Conduction Computation for Thermal Models. Journal of Hydraulics Division, ASCE, Vol.103, HY10. [The exchange of energy between water and a stream or reservoir bed is widely discussed]

Mauersberger P. (1982) General Principles in Deterministic Water Quality Modeling, in *Mathematical modeling of Water Quality: Streams, Lakes, and Reservoirs,* edited by Orlob G.T. pp.42 – 115. John Wiley & Sons, Chichester. [It gives fundamentals for mathematical modeling of water quality]

Stolzenbach K.D. (1976) Heat Exchange at the Water Surface. Lecture Notes for the International Advance Course, Dubrownik, Yugoslavia. [A detailed study of heat exchange between water and the atmosphere]

White, F. M. (1988). *Heat and Mass Transfer*. University of Rhode Island, Reading. [It is a monograph on heat and mass transfer]

Biographical Sketch

W. Czernuszenko, Professor, Institute of Geophysics, Polish Academy of Sciences, Warsaw, Poland.He has graduated from the Faculty of Civil Engineering of the Warsaw University of Technology (M.Sc. Degree) as well as from the Faculty of Applied Mathematics of the Warsaw University, also with M.Sc. degree in Mathematics. He received his Ph D degree from the Warsaw University of Technology upon presentation of the doctorate thesis "Mass transfer in the open channel flow " and his Dr. Hab. Degree based on his monograph "Dispersion of Pollutants in Rivers and Channels. In 1991 – 1994, he occupied the position of visiting professor at Mechanical Department of the University of Mississippi and he worked for National Center for Computational Hydroscience and Engineering at the same University.

As a researcher he is interested in application of fluid mechanics to environmental problems related to mass and heat transfer in open channel flows and particularly to modeling of pollution transport and sediments in rivers. He has been doing experimental works in fields and water laboratories related to turbulence in rivers and open channels, mixing processes in flowing surface waters and fluvial hydraulics. Also, he has been doing some theoretical works on mathematical modeling of free surface flows, modeling of turbulence and transport of mass in turbulent flows.

The results of his works have been published in many scientific international journals and presented at some international conferences.